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# Are convertible bonds underpriced? An analysis of the French market

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**Abstract**

We investigate the pricing of convertible bonds on the French convertible bond market using daily market prices for a period of 18 months. Instead of a firm-value model as used in previous studies, we use a stock-based binomial-tree model with exogenous credit risk that accounts for all important convertible bond specifications and is therefore well suited for pricing convertible bonds. The empirical analysis shows that the theoretical values for the analyzed convertible bonds are on average more than 3% higher than the observed market prices. This result applies to both the standard convertibles and the exchangeable bonds in our sample. The difference between market and model prices is greater for out-of-the-money convertibles than for at- or in-the-money convertibles. A partition of the sample according to maturity indicates that there is a positive relationship between underpricing and maturity with decreasing mispricing for bonds with shorter time to maturity.


*JEL classification:* G13; G15
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## 1. Introduction

Convertible bonds are complex and widely used [1] financial instruments combining the characteristics of stocks and bonds. The possibility to convert the bond into a predetermined number of stocks offers participation in rising stock prices with limited loss potential, given that the issuer does not default on its bond obligation. Convertible bonds often contain other embedded options such as call and put provisions. These options can be specified in various different ways, further adding to the complexity of the instrument. Especially, conversion and call opportunities may be restricted to certain periods or stock price conditions and the call price may vary over time.

The purpose of this study is to investigate whether prices observed on secondary markets are below the theoretical fair values (obtained by a contingent claims pricing model), as is believed by many practitioners (see, for example, Noddings et al., 1998).

Theoretical research on convertible bond pricing was initiated by Ingersoll (1977a) and Brennan and Schwartz (1977), who both applied the contingent claims approach to the valuation of convertible bonds. In their valuation models, the convertible bond price depends on the firm value as the underlying variable. Brennan and Schwartz (1980) extend their model by including stochastic interest rates. However, they conclude that the effect of a stochastic term structure on convertible bond prices is so small that it can be neglected for empirical purposes. McConnell and Schwartz (1986) develop a pricing model based on the stock value as stochastic variable. To account for credit risk, they use an interest rate that is grossed up by a constant credit spread. Noting that credit risk of a convertible bond varies with respect to its moneyness, Bardhan et al. (1993) and Tsiveriotis and Fernandes (1998) propose an approach that splits the value of a convertible bond into a stock component and a straight bond component. Buchan (1998) extends the Brennan and Schwartz (1980) model by allowing senior debt and implements a Monte Carlo simulation approach to solve the valuation equation.

Despite the large size of international convertible bond markets, very little empirical research on the pricing of convertible bonds has been undertaken. Previous research in this area was performed by King (1986), who finds that, for a sample of 103 American convertible bonds, a slight underpricing of 3.75% exists on average, i.e., market prices are 3.75% below model prices. Using monthly price data, Carayannopoulos (1996) empirically investigates 30 American convertible bonds for a one-year period beginning in the fourth quarter of 1989. Using a convertible bond valuation model with Cox et al. (1985) stochastic interest rates, he finds a larger mean underpricing of 12.9%. As King (1986), he reports that deep out-of-the-money bonds are underpriced, at- or in-the-money bonds are slightly overpriced. Buchan (1997) implements a firm-value model using also a CIR term structure model. In contrast to the

[1] The Bank for International Settlements reports an outstanding amount of international convertible bonds of 223.6 billion US dollars (not including domestic issues) per December 2000 (BIS, 2001).

above mentioned studies, she finds that, for 35 Japanese convertible bonds, [2] model prices are slightly below observed market prices on average by 1.7%.

A drawback of these previous pricing studies is the small number of data points per convertible bond: Buchan (1997) tests her pricing models only for one calendar day (bonds priced per March 31, 1994), King (1986) for two days (bonds priced per March 31, 1977, and December 31, 1977), and Carayannopoulos (1996) for 12 days (one year of monthly data). Our study does not suffer from this limitation because we use 18 months of daily price data, ranging from February 19, 1999 to September 5, 2000.

Furthermore, we undertake the first pricing study for the French convertible bond market. We examine the French market for convertible bonds because of the availability of accurate daily market prices, its large size compared to other European markets, the high ratio of domestic issues and the liquidity of many of its bond issues. Additionally, all bonds are exchange traded. In terms of the outstanding amount of convertible bond issues as well as the number of issues, the French market is the largest in Europe. [3] Furthermore, among the 10 largest outstanding convertible bond issues in Europe at the beginning of 2000, five were issued by French companies and all of them had a volume of more than 1.5 billion euros. Our sample includes the 21 most liquid convertible bonds in the French market, 14 of them with an issue volume in excess of 500 million euros. The French convertible bond market is one of the longest established convertible bond markets in Europe and is characterized by the presence of a relatively large base of private investors, convertible bond funds, and hedge-fund activity. [4] Although French convertible bonds differ from other convertible bonds because they are usually issued with a price and conversion value equal to par and entail a redemption premium, the valuation problem is not exacerbated by this feature. The French convertible bond market can therefore be considered well suited for a representative pricing test.

Furthermore, our study contributes the first empirical test of a convertible bond pricing model based on the direct modeling of the stock price, as proposed by McConnell and Schwartz (1986), instead of using a firm-value model. Whereas a stock price-based model can easily be estimated with standard methods, the fact that firm values are not observable makes them notoriously hard to calibrate. [5] Extending the approaches by McConnell and Schwartz (1986) and Tsiveriotis and Fernandes (1998) to be able to account for the complex bond characteristics such as embedded call features with various trigger conditions, we implement a binomial-tree model with exogenous credit risk. The convertible bond prices generated by the binomial-tree model are compared to the market prices of the investigated

[2] All but one bond were out-of-the-money on March 31, 1994.

[3] As of December 1999, while the outstanding amount was \$41 billion in France, it was only \$21.3 billion in the UK and \$18.2 billion in Germany. See Hope (2000) for more detailed market statistics.

[4] Noddings et al. (1998) provide further details on the French convertible bond market.

[5] The practical problems associated with firm-value models are discussed in several articles on credit risk modeling, such as Jarrow et al. (1997).

convertible bonds. Two other approaches, a simple component model and an exchange-option model are also implemented and serve as very simple reference models.

On average, an underpricing of more than 3% is detected. This result holds for both exchangeables and standard convertibles. For a few convertible bonds, overpricing can be observed. A partition of the sample according to the moneyness indicates that the underpricing decreases for convertible bonds that are further in-the-money. Comparing the degree of underpricing to the maturity of the convertible bonds, we find that, the longer the maturity, the lower is the market price observed relative to the price generated by the model.

The paper is organized as follows: First, we discuss convertible bond pricing models and introduce the model used in the empirical investigation. Second, we describe the data set and discuss the specific characteristics of the convertible bonds examined. Finally, we present results of the empirical study comparing theoretical model prices with observed market prices.

## 2. Pricing models for convertible bonds

### 2.1. Component and margrabe models

In practice, the traditionally used method for pricing convertible bonds is the component model, also called the synthetic model. [6] This method separates the convertible bond into a straight bond component and a call option. The fair value of the two components can be calculated with standard formulas. The value of the option has traditionally been computed with the Merton (1973) and Black and Scholes (1973) option pricing formula. Such a model is therefore straightforward to implement and entails very low computational cost.

Unlike call options, where the strike price is known in advance, convertible bonds contain an option component with a stochastic strike price. It is stochastic because the value of the bond to be delivered in exchange for the shares is usually not known in advance unless conversion is certain not to occur until maturity. The future strike price depends on the future development of interest rates and the future credit spread. This problem is addressed by pricing the conversion option as an option to exchange one asset for another. Thus, the convertible bond is viewed as the sum of a straight bond plus an option giving the holder the right to exchange the straight bond for a certain amount of stocks. Margrabe (1978) first presented a closed-form solution for exchange options. We therefore refer to the exchange-option approach as the Margrabe model. Using the original Margrabe (1978) formula for valuing the exchange option implicitly assumes geometric Brownian motion as the underlying price process for the straight bond. However, geometric Brownian motion is generally not considered an appropriate process specification for bond prices. [7]

[6] See, for example, Connolly (1998).

[7] For example, mean-reversion and the maturity dependence of bond price volatility is not reflected by geometric Brownian motion.

Using the closed form pricing formulae by Merton (1973), Black and Scholes (1973) and Margrabe (1978) for the valuation of the conversion option entails some further serious drawbacks. For example, these formulae refer to European-style options whereas almost all convertible bonds can be exercised prior to maturity. [8] Most importantly, component models neglect the presence of embedded call and put features. While embedded put options tend to be fairly rare, most convertible bonds can be called by the issuer.

### 2.2. Binomial-tree model with exogenous credit risk

#### 2.2.1. Specifying the binomial tree

Because of the drawbacks of the traditional pricing models, we implement as a third and most precise approach a binomial-tree model with exogenous credit risk that is able to account for embedded options and early exercise. We construct the univariate binomial tree with 100 time steps following Cox et al. (1979). The binomial tree is based on the stock price as described in McConnell and Schwartz (1986). State-dependent credit risk is incorporated using the approach by Tsiveriotis and Fernandes (1998). To be able to account for the various complex characteristics of the bonds in our sample such as embedded options and triggers, we extend the aforementioned approaches with several contract-specific boundary conditions.

The terminal condition is given by

$$\Omega_T = \mathrm{Max}(n_T S_T, \kappa N),$$

where $\Omega_T$ is the fair value of the convertible bond at maturity $T$, $n_T$ is the conversion ratio, i.e. the number of stocks the bond can be exchanged for, $S_T$ is the equity price (underlying) at time $T$, $\kappa$ is the final redemption ratio at time $T$ in percentage points of the face value, and $N$ is the face value of the convertible. The expression $n_T S_T$ can be interpreted as the conversion value. This condition is considered for all endnodes in the tree.

Due to the American character of the instrument, it is necessary to check the following three boundary conditions in each node of the tree.

The *conversion boundary condition* implies that

$$\Omega_t \geqslant n_t S_t \quad \forall\, t \in [\tau_\Gamma, T_\Gamma]. \tag{1}$$

During the conversion period starting at $\tau_\Gamma$ and ending at $T_\Gamma$, the value of the convertible bond cannot be less than the conversion value; otherwise, an arbitrage opportunity would exist.

The *call boundary condition* states that, when the conversion ratio is higher than the trigger $\Xi_t$, i.e. the trigger condition $n_t S_t > \Xi_t$ is satisfied,

[8] As long as the coupon rate is less than the dividend yield, this is not a problem. As Subrahmanyam (1990) points out, it is suboptimal to exercise a Margrabe (1978) exchange-option prior to maturity if there is a "yield advantage", i.e., the cash flows of the exchangeable instrument are greater than the cash flows of the obtained asset at each point.

$$\Omega_t \leqslant \mathrm{Max}(K_t + \Theta_t, n_t S_t) \quad \forall\ t \in [\tau_K, T_K] \tag{2}$$

must hold. $K_t$ is the relevant early redemption price (call price) at time $t$. The call period starts at $\tau_K$ and ends at $T_K$. $\Theta_t$ is a safety premium that accounts for the empirical fact, described by Ingersoll (1977b), that the issuer usually does not call immediately when $K_t$ is triggered. Firms may want the conversion value to exceed the call price by a certain amount to ensure it will still exceed the call price at the end of the call notice period, which is normally three months in the French market. The safety premium is set equal to zero in this study, resulting in a conservative valuation of the convertible bonds. The price of a convertible bond cannot, at the same time, be higher than the conversion value and higher than the call price. If such a situation occurred, the issuer could realize arbitrage gains by calling the convertible bond.

The *put boundary* condition requires that

$$\Omega_t \geqslant p_t, \quad \forall\ t \in [\tau_p, T_p]. \tag{3}$$

$p_t$ is the relevant put price at time $t$. If the convertible price were below the relevant put price, the investor could exercise the put option and realize a risk-free gain. Since put features are absent in our sample of convertible bonds, the put boundary condition does not affect the results of this analysis.

In each node, it is necessary to check whether each boundary condition is satisfied and to determine the implications on the value of the convertible bond with respect to the optimal calling behavior of the issuer and the optimal conversion behavior of the investor. Ingersoll (1977a) provides a discussion on the optimal call and conversion policy.

Fig. 1 shows a computationally efficient way of checking the validity of the boundary conditions and the effects on the convertible bond. There are four possible outcomes: The convertible bond continues to exist without being called or converted. Alternatively, it may be called by the issuer, converted by the holder, or called by the issuer and subsequently converted by the investor. The last scenario is often called *forced conversion* because the investor is induced to convert exclusively by the fact that the issuer has called the bond.

### 2.2.2. Integration of credit risk

The classical convertible bond pricing articles of Ingersoll (1977a) and Brennan and Schwartz (1977, 1980) use the firm value as a stochastic variable. According to this approach, credit risk is modeled endogenously by assuming that default occurs when the firm value falls below the value of the debt. As noted by Jarrow et al. (1997), firm-value models are hard to implement in practice because the firm value is not observable and even the firms' liabilities often cannot be observed. Not knowing the debt value distorts the volatility estimation of the firm value and the exact time of default.

McConnell and Schwartz (1986) present a pricing model based on the stock value as stochastic variable. To account for credit risk, they use an interest rate that is "grossed up to capture the default risk of the issuer" rather than the risk-free rate.

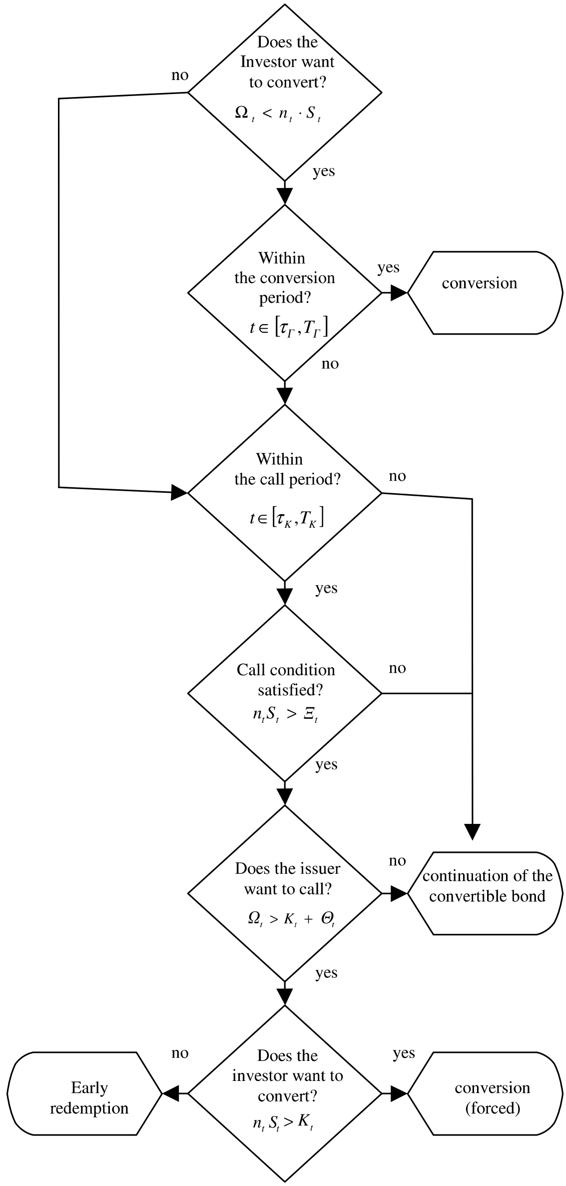


Fig. 1. Flow chart of optimal option exercise. This flow chart presents the decision procedure performed in each node of the tree according the boundary conditions.

Since they implicitly use a constant credit spread, they do not consider that the credit risk of a convertible bond varies with respect to its moneyness.

For this reason, Bardhan et al. (1993) and Tsiveriotis and Fernandes (1998) propose an approach that splits the value of a convertible bond into a stock component and a straight bond component. These two components belong to different credit risk categories. The former is risk free because a company is always able to deliver its own stock. The latter, however, is risky because coupon and principal payments depend on the issuer's capability of distributing the required cash amounts. It is straightforward to discount the stock part of the convertible with the risk-free interest rate and the straight bond component with a risk-adjusted rate. When the convertible bond is deep in the money, its value should be discounted using the risk-free rate. When the bond is deep out of the money, the straight bond component is very high and so is its defaultable part. This method is an improvement over the approach of McConnell and Schwartz (1986) because it clearly identifies the defaultable part of the convertible and thus its credit risk exposure. We therefore adopt this approach in incorporating a constant exogenous credit spread into our binomial-tree model. The appropriate credit spread is given by the difference between the yield to maturity of a straight bond of the company and the yield to maturity of a risk-free sovereign bond. The bonds have to be comparable, i.e. they must have similar seniority, coupon and maturity. If no straight bond comparable to the convertible exists, the credit spread can be estimated using the rating of the issuing firm.

## 3. Data

### 3.1. Convertible bonds

We consider French convertible bonds outstanding as of September 5, 2000. Daily convertible bond prices as well as the corresponding synchronous stock prices are available from February 19, 1999, through September 5, 2000. They were provided by Mace Advisers. To exclude illiquid issues from the sample, we require every issue to satisfy three conditions cumulatively. [9] First, we exclude from the sample all convertibles with a market capitalization below USD 75 million. Second, all issues have to have a minimum average exchange-based trading volume for the last two quarters of at least USD 75 million. Third, we consider a convertible only if at least three market makers out of the top 10 convertible underwriters quote prices with a bid/ask spread not larger than two percentage points. In addition, cross-currency convertibles are excluded from the sample. As a result, our convertible bond universe consists of 21 French franc/euro-denominated issues with a total of 6662 data points. Table 1 gives an overview of the analyzed convertible bonds. In the sample, there are seven exchangeable bonds. In these cases, the firm issuing the bond and the firm issuing the stock into which the bond can be converted are not identical.

Table 2 summarizes the detailed contractual specifications that are extracted from the official and legally binding "offering circulars". This proved to be necessary because almost every electronic database tends to suffer from an over-standardization

[9] These requirements are the same that UBS Warburg uses as exit criteria for its convertible bond index family.

Table 1
Specification of the convertible bonds

| Convertible into shares of | Issuing company | Sector of issuing firm | Maturity | Coupon (%) | Exchange-able bonds |
|---|---|---|---|---|---|
| Axa | Finaxa | Holdings | 2007 | 3.00 | X |
| Axa | Suez Lyonnaise des Eaux | Services | 2004 | 0.00 | X |
| Axa | Axa | Insurance | 2014 | 2.50 | |
| Axa | Axa | Insurance | 2017 | 3.75 | |
| Bouygues | Bouygues | Telecommunications | 2006 | 1.70 | |
| Bull | Bull | Information technology | 2005 | 2.25 | |
| Carrefour (Promodès) | Carrefour (Promodès) | Retail stores | 2004 | 2.50 | |
| France Télécom | France Télécom | Telecommunications | 2004 | 2.00 | |
| Infogrames Entertainment | Infogrames Entertainment | Entertainment | 2005 | 1.50 | |
| LVMH | Financière Agache | Consumption goods | 2004 | 0.00 | X |
| Peugeot | Peugeot | Automobiles | 2001 | 2.00 | |
| Pinault-Printemps-Redoute | Artémis | Holdings | 2005 | 1.50 | X |
| Pinault-Printemps-Redoute | Pinault-Printemps-Redoute | Retail stores | 2003 | 1.50 | |
| Rhodia | Aventis (Rhône Poulenc) | Pharmaceuticals | 2003 | 3.25 | X |
| Scor | Scor | Insurance | 2005 | 1.00 | |
| Société Générale | Société Vinci Obligations[a] | Insurance | 2003 | 1.50 | X |
| Total Fina | Belgelec Finance[b] | Energy/services | 2004 | 1.50 | X |
| Usinor | Usinor | Steel | 2006 | 3.00 | |
| Usinor | Usinor | Steel | 2005 | 3.88 | |
| Vivendi | Vivendi | Entertainment/services | 2004 | 1.25 | |
| Vivendi | Vivendi | Entertainment/services | 2005 | 1.50 | |

This table gives an overview of the analyzed convertible bonds with maturity and coupon information as well as the business sector of the issuing company. Exchangeable bonds are denoted by X.

[a] Société Vinci Obligations is a wholly-owned subsidiary of CUF. The indicated sector refers to CUF.

[b] Belgelec Finance is a special purpose vehicle controlled by Tractebel. The indicated sector refers to Tractebel.

syndrome. Although most bonds in our sample have very similar specifications, some contractual provisions are so specific that they can hardly be collected in predefined data types. Several convertibles in our sample are "premium redemption" convertibles, i.e. the redemption at maturity is above par value. In this case, the final redemption is given by $\kappa N$ with the final redemption ratio $\kappa > 1$. Twenty of the 21 analyzed convertibles include a call option, allowing the issuer to repurchase the bond for a certain price $K_t$, called "call price" or "early redemption price". This price can vary over time. Usually, the call price $K_t$ is determined in such a way that the holder of the bond obtains an equal or similar return as when holding the convertible bond until maturity without converting. For almost all examined convertibles, early

Table 2
Specification of embedded options

| Issuing company | Initial conversion ratio | Final redemption ratio (%) | Call | Call trigger ratio (%) | Call trigger basis | Maximum issue volume | Green shoe option (%) |
|---|---|---|---|---|---|---|---|
| Finaxa | 1 | 118.18 | No | – | – | 1704 | 40.00 |
| Suez | 1 | 109.83 | Yes | – | – | 787 | 0.00 |
| Axa | 1 | 139.93 | Yes | 125 | Redemption | 1524 | 13.04 |
| Axa | 1 | 162.63 | Yes | 125 | Redemption | 1265 | 13.04 |
| Bouygues | 1 | 100.00 | Yes | 115 | Redemption | 500 | 8.00 |
| Bull | 1 | 116.60 | Yes | 120 | Redemption | 181 | 13.02 |
| Carrefour | 1 | 100.00 | Yes | 120 | Redemption | 589 | 0.00 |
| France Télécom | 10 | 100.00 | Yes | 115 | Face value | 2031 | 9.85 |
| Infogrames Entertainment | 1 | 118.23 | Yes | 250[a] | Redemption | 401 | 13.04 |
| Financière Agache | 1 | 111.77 | Yes | 120 | Face value | 500 | 0.00 |
| Peugeot | 1 | 123.64 | Yes | 100 | Redemption | 604 | 0.00 |
| Artémis | 10 | 110.54 | Yes | 120 | Face value | 457 | 0.00 |
| Pinault | 1 | 103.63 | Yes | 130 | Redemption | 1000 | 8.00 |
| Aventis | 1 | 100.00 | Yes | 130 | Redemption | 1014 | 0.00 |
| Scor | 1 | 112.55 | Yes | 120 | Redemption | 233 | 13.04 |
| Vinci | 1 | 100.00 | Yes | 130 | Redemption | 425 | 0.00 |
| Belgelec Finance | 1 | 100.00 | Yes | 130 | Redemption | 1266 | 0.00 |
| Usinor | 1 | 110.91 | Yes | 125 | Redemption | 381 | 12.00 |
| Usinor | 1 | 100.00 | Yes | 130 | Redemption | 497 | 12.00 |
| Vivendi | 1 | 100.00 | Yes | 115 | Redemption | 1700 | 11.76 |
| Vivendi | 1 | 106.27 | Yes | 115 | Redemption | 3000 | 13.33 |

The *call trigger ratio* can refer to either the face value of the convertible or to the early redemption price (denoted as *redemption*). The *maximum issue volume* indicates the highest amount of bonds issued according to the offering circular (including green shoe, if present) measured in million euros. The *green shoe option* indicates the percentage of the maximum issue volume that could be issued on a discretionary basis. Zero means that no green shoe was present.

[a] After July 1, 2003, the call trigger is reduced to 125% of the early redemption price.

redemption is restricted to a certain predetermined period from $\tau_K$ to $T_K$. The period during which callability is not allowed is called the ''call protection period''. An additional restriction to callability in form of a supplementary condition to be satisfied is given by the ''call condition''. Callability is only allowed if the parity $n_t S_t$ exceeds a ''call trigger'' $\Xi_t$. [10] The call trigger is calculated as a percentage of either the early redemption price or the face value. The last column in Table 2 shows, for each bond, which of the two methods applies. If the trigger feature is present, the callability is called ''provisional'' or ''soft'' call, if it is absent, the callability is ''absolute'' or ''unconditional''. For almost all convertibles, the trigger feature is present. Only the

[10] The exact contractual specification of the call condition often states that the inequality $n_t S_t > \Xi_t$ must hold for a certain time (often 30 days) before the bond becomes callable. This ''qualifying period'' introduces a path dependent feature not considered in the analysis.

bond issued by Suez Lyonnaise des Eaux lacks a trigger and has an unconditional callability. Another special case is Infogrames Entertainment, which has an unusual time-varying call trigger: Within the period from May 30, 2000 to June 30, 2003, the call trigger is set at 250% of the early redemption price. After July 1, 2003, the call trigger is reduced to 125% of the early redemption price.

Usually, the conversion ratio $n_t$ is constant over time. It changes in case of an alteration of the nominal value of the shares (stock subdivisions or consolidations), extraordinary dividend payments and other financial operations that directly affect the stock price. Conversion is possible within a certain period, called conversion period. The conversion period starts at time $\tau_\Gamma$ and ends at time $T_\Gamma$. For all the issues in our sample, the end of the conversion period coincides with the maturity of the convertible bond, i.e. $T_\Gamma = T$.

Table 3 presents performance measures of the convertible bonds compared to the underlying stock. The return of the underlying stock is consistently higher (lower)

Table 3
Performance measures of the convertible bonds and the underlying stock

| Convertibles | Exchangeable bonds | Underlying stock | | Convertible bond | |
|---|---|---|---|---|---|
| | | Mean | Volatility | Mean | Volatility |
| Axa 2007 | X | 0.194 | 0.321 | 0.136 | 0.339 |
| Axa 2004 | X | 0.232 | 0.320 | 0.107 | 0.241 |
| Axa 2014 | | 0.194 | 0.321 | 0.034 | 0.198 |
| Axa 2017 | | 0.393 | 0.357 | 0.281 | 0.353 |
| Bouygues | | 0.750 | 0.495 | 0.628 | 0.502 |
| Bull | | −1.311 | 0.627 | −0.452 | 0.265 |
| Carrefour | | 0.053 | 0.433 | 0.049 | 0.353 |
| F. Télécom | | 0.369 | 0.518 | 0.298 | 0.479 |
| Infogrames | | 0.131 | 0.624 | 0.118 | 0.287 |
| LVMH | X | 0.483 | 0.342 | 0.363 | 0.281 |
| Peugeot | | 0.342 | 0.315 | 0.093 | 0.264 |
| Pinault 2005 | X | 0.206 | 0.336 | 0.033 | 0.169 |
| Pinault 2003 | | 0.243 | 0.340 | 0.078 | 0.310 |
| Rhodia | X | −0.038 | 0.305 | 0.027 | 0.156 |
| Scor | | 0.067 | 0.397 | 0.037 | 0.220 |
| S. Générale | X | 0.470 | 0.354 | 0.137 | 0.180 |
| Total Fina | X | 0.279 | 0.347 | 0.136 | 0.180 |
| Usinor 2006 | | 0.043 | 0.413 | 0.007 | 0.366 |
| Usinor 2005 | | −0.348 | 0.404 | −0.048 | 0.186 |
| Vivendi 2004 | | 0.126 | 0.385 | 0.034 | 0.295 |
| Vivendi 2005 | | 0.185 | 0.397 | 0.124 | 0.337 |
| Mean total | | 0.146 | 0.398 | 0.106 | 0.284 |
| Mean exchangeables | | 0.261 | 0.332 | 0.134 | 0.221 |
| Mean straight convertibles | | 0.088 | 0.430 | 0.092 | 0.315 |

This table presents mean and volatility of the convertible bonds and the respective underlying stock for the period during which the pricing was investigated. All values are continuously compounded. Exchangeable bonds are denoted by X.

than that of the convertible bonds for positive (negative) stock returns. During the examination period, mean stock volatilities were higher than the corresponding values of convertible bonds. These observations are consistent with the hybrid nature of convertibles.

We examined whether firms issuing standard convertibles have different characteristics than those issuing exchangeables. We analyzed capital structure (leverage) and business sector and found that the issuers in our sample differed widely with respect to such characteristics. However, there seems to be no apparent pattern regarding the firms' preferences for standard convertibles or exchangeables. Because the small number of issues in the sample (seven exchangeable bonds) would impede any statistically conclusive findings, we did not undertake a more detailed analysis of this issue.

### *3.2. Input parameters*

All interest rate data is obtained from Primark Datastream. For interest rates of one year or less (seven days, 1, 2, 3, 6, 12 months), we use Eurofranc rates. For longer maturities (1–10 years), we extract spot rates from swap rates. We observed that the one-year Eurofranc rate was systematically lower than the corresponding one-year swap rate. Under the assumption that the Eurofranc rates represent a better proxy for the theoretical credit risk-free rates, we adjust down the swap-extracted term structure by the difference between the one-year Eurofranc rate and the one-year swap rate. Furthermore, we use linear interpolation to obtain the complete continuous term structure of spot rates.

Besides directly observable input parameters, such as stock prices and interest rates, the pricing models require input parameters that have to be estimated and thus are a source of estimation error. These variables include volatility, dividends, and credit spreads. Summary statistics of these input parameters are presented in Table 4.

The most important input parameter to be estimated is the volatility of the underlying stock price. Research on stock volatility estimation is plentiful. A popular approach is the implied volatility concept. With option pricing formulas, it is possible to extract market participants' volatility estimations from at-the-money option prices. However, most liquid options have shorter maturities than convertibles. We therefore estimate volatility on a historical basis. The relevant volatility is calculated as the standard deviation of the returns of the last 520 trading days, corresponding to two trading years (two times 260 trading days). [11]

We model future dividends using a constant absolute cash flow based on historical dividends obtained from Primark Datastream. Without adjustments, this approach is not computationally feasible because it implies a non-recombining binomial tree

[11] Obviously, the results depend on the length of the rolling time window used for the calculation. However, we found our results to be fairly robust with respect to this time window. For example, calculating the historical volatility based on a one year rolling window as Carayannopoulos (1996), we find the average pricing error to deviate less than 10% (29 basis points) from the error reported in Table 5.

Table 4
Statistics of the input parameters used

| Convertibles | Mean of the input | | Mean credit spread (in basis points) |
|---|---|---|---|
| | Volatility (%) | Dividend yield (%) | |
| Axa 2007 | 35.85 | 2.18 | 45 |
| Axa 2004 | 35.89 | 2.19 | 40 |
| Axa 2014 | 35.85 | 2.18 | 73 |
| Axa 2017 | 36.58 | 2.16 | 74 |
| Bouygues | 46.08 | 1.00 | 84 |
| Bull | 66.05 | 0.00 | 300 |
| Carrefour | 37.58 | 0.97 | 42 |
| F. Télécom | 46.45 | 1.69 | 31 |
| Infogrames | 56.16 | 0.00 | 300 |
| LVMH | 39.95 | 1.62 | 80 |
| Peugeot | 39.61 | 1.89 | 40 |
| Pinault 2005 | 38.99 | 1.29 | 100 |
| Pinault 2003 | 38.93 | 1.29 | 80 |
| Rhodia | 45.07 | 4.09 | 59 |
| Scor | 39.12 | 5.75 | 26 |
| S. Générale | 45.04 | 3.94 | 50 |
| Total Fina | 39.29 | 2.79 | 50 |
| Usinor 2006 | 45.66 | 5.46 | 124 |
| Usinor 2005 | 44.86 | 5.46 | 119 |
| Vivendi 2004 | 32.06 | 1.98 | 75 |
| Vivendi 2005 | 32.37 | 2.00 | 82 |

The input volatility of the underlying stock is a two year historical average. The dividend yield is the mean percentage dividend over the observation period. The mean credit spread is calculated from the credit spread input time series.

for the stock price. [12] To allow for a recombining tree, we implement the approximation method proposed by Hull and White (1988). This method separates the stock price process into a stochastic stock component adjusted for the present value of future dividends and a deterministic dividend component. [13]

In Table 4, the mean credit spread [14] is expressed in basis points over the relevant period. Where the issuer has straight debt in the market, the credit spread is calculated on the basis of the traded yield spread. Otherwise, it is calculated on the basis of credit spread indices, e.g. the Bloomberg Fair Market Curves and UBS Credit Indices, according to the characteristics of the sector in the relevant rating category.

[12] Whereas the number of endnodes for a recombining tree grows linearly with the number of steps in the tree, it grows exponentially for a non-recombining tree.

[13] Alternatively, dividends can be modeled using a constant dividend yield. This method does not present the problem of non-recombining trees. However, a constant dividend yield implies dividends that co-move with the stock price. Because companies tend to smooth dividend payments, this assumption is only realistic in the very long run. Still, we tested this procedure and found only a small effect on the results.

[14] Credit spread time series were provided by UBS Warburg.

Table 5
Pricing overview for the binomial-tree model

| Convertibles | Data points | Mean percentage overpricing (%) | Root mean squared error | Exchangeable bonds |
|---|---|---|---|---|
| Axa 2007 | 402 | −2.98*** | 0.034 | X |
| Axa 2004 | 396 | −1.06*** | 0.021 | X |
| Axa 2014 | 402 | −5.23*** | 0.061 | |
| Axa 2017 | 149 | −11.02*** | 0.111 | |
| Bouygues | 402 | −1.76*** | 0.041 | |
| Bull | 89 | −14.07*** | 0.142 | |
| Carrefour | 256 | 1.18*** | 0.030 | |
| F. Télécom | 402 | −1.96*** | 0.043 | |
| Infogrames | 78 | −2.95*** | 0.033 | |
| LVMH | 376 | −5.14*** | 0.059 | X |
| Peugeot | 402 | 2.43*** | 0.035 | |
| Pinault 2005 | 402 | −4.18*** | 0.045 | X |
| Pinault 2003 | 320 | −2.53*** | 0.037 | |
| Rhodia | 232 | −6.76*** | 0.078 | X |
| Scor | 320 | −0.19** | 0.019 | |
| S. Générale | 402 | −2.65*** | 0.030 | X |
| Total Fina | 315 | −2.78*** | 0.030 | X |
| Usinor 2006 | 402 | 3.49*** | 0.042 | |
| Usinor 2005 | 149 | −9.09*** | 0.092 | |
| Vivendi 2004 | 402 | −0.80*** | 0.019 | |
| Vivendi 2005 | 364 | −0.05 | 0.019 | |
| Mean total | | −3.24*** | | |
| Mean exchangeables | | −3.65*** | | |
| Mean straight convertibles | | −3.04*** | | |

*Data points* indicates the number of days for which model prices are computed. ***, ** and * denote significance levels of 1%, 5% and 10%, respectively, for the rejection of the null hypothesis that model and market prices are equal in the mean. The *root mean squared error* is the non-central standard deviation of the relative deviations of model prices from market prices. Exchangeable bonds are denoted by X.

## 4. Results

The observed convertible bond prices on the French market are compared with theoretical prices obtained with the binomial-tree model. The main results are summarized in Table 5. In analogy to the methodology used by Sterk (1982) and others who tested option pricing models, the table provides data about the mean percentage overpricing of each issue. The overpricing is presented for each convertible bond as an average of the deviation between the theoretical and observed price for each observation. A negative value indicates an observed underpricing, i.e., the theoretical value is above the observed market price. The fourth column shows the root mean squared error of the relative mispricing. The RMSE shows the non-central standard deviation of the relative deviations of model prices from market prices. It can be interpreted as a measure for the pricing fit of the model relative to market prices.

The binomial-tree model exhibits an average underpricing of 3.24%, i.e., market prices are lower than our model prices. [15] For comparison, we also computed the overall underpricing average for the component and the Margrabe models. The corresponding underpricing for the component and Margrabe models (not displayed in the table) amounts to 8.74% and 5.60%, respectively. The much larger underpricing compared to the binomial model is an obvious consequence of the fact that these models do not account for the call feature, which is present in all but one of the examined convertible bonds. Callability reduces the stock-driven upside-potential and thus has a negative impact on convertible prices. This result demonstrates the importance of modeling embedded options when valuing convertible bonds.

The underpricing of 3.24% for the binomial model prevails even though we value the convertible bond conservatively by setting the safety premium to zero. A partition of the sample into exchangeables and standard convertibles indicates an underpricing of more than 3% for both classes. The underpricing difference between the two classes is relatively small: The average underpricing of the exchangeables is 3.65% while the average underpricing of the standard convertibles is only 3.04%. This relatively small difference in valuation prevails despite the rather different risk-return characteristics (see Table 3) of the bonds and the underlying stocks between the two subsamples. In other words, there seems to be no fundamental difference in valuation between standard convertible bonds and exchangeable bonds.

In the entire sample, the binomial-tree model detects three cases of overpricing. Among those cases, the overpricing amounts to 1.18% for Carrefour 2004, 2.43% for Peugeot 2001, and 3.49% for Usinor 2006. In contrast, 18 convertible bonds show a mean percentage underpricing. The significance test indicates that the mean price deviation of 16 of them is different from zero at a 1% significance level. Our results are very similar to those obtained by King (1986), who finds a significant mean underpricing of the same order of magnitude (3.75%). While Carayannopoulos (1996) reports a substantial underpricing of 12.9%, Buchan (1997) finds a slight overpricing of 1.7%, although it is not statistically significant. However, those previous results are all derived from a very small number of data points per convertible bond. Moreover, they are obtained using firm-value models, which are inherently difficult to parameterize because the firm value is not observable.

Fig. 2 shows overpricing of each daily observed market price measured by the binomial-tree model with respect to the moneyness of the bond. The moneyness is estimated by dividing the parity through the investment value. Parity is the value of shares that can be obtained by converting the bond. The investment value denotes the value of the convertible bond under the hypothetical assumption that the conversion option does not exist. The relationship between overpricing and moneyness is non-linear. In the mean, the market significantly underprices bonds that are at-the-money and out-of-the-money and slightly overprices in-the-money convertibles relative to model prices. However, the standard deviation is rather large, as can be

[15] Clearly, market mispricing can only be observed with respect to a pricing model. Our analysis can therefore be viewed as test of a pricing model.

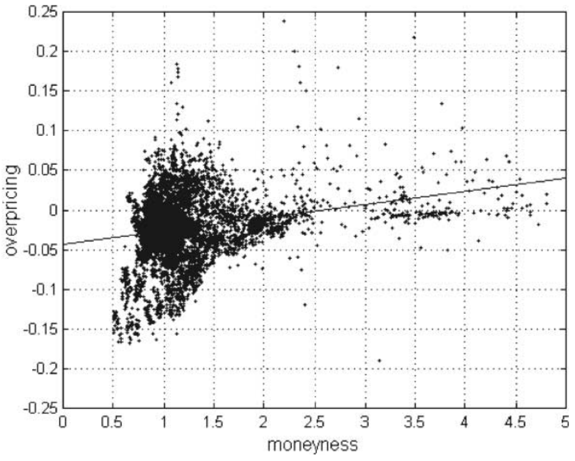


Fig. 2. Moneyness/overpricing relationship for the binomial-tree model. This graph shows the percentage overpricing of each daily observed market price measured by the binomial-tree model with respect to the moneyness of the bond. The moneyness is estimated by dividing the conversion value through the investment value. The conversion value is the value of the shares that can be obtained by converting the bond. The investment value denotes the value of the convertible bond under the hypothetical assumption that the conversion option does not exist.

seen in Table 6. The mispricing in the mean disappears as convertibles move deeply in-the-money. A possible explanation for this effect is that, for deep in-the-money convertibles, the probability of conversion is very high and the time value of the conversion option becomes very small. Therefore, they have to trade very close to parity. For this reason, they are easier to price than at-the-money convertibles where the time-value component of the conversion option is much larger. Both King (1986) and Carayannopoulos (1996) also report a negative relationship between underpricing and moneyness. In the case of Carayannopoulos (1996), the effect is even more pronounced than in this study.

Fig. 3 shows a slight relationship between overpricing and maturity. The longer the time to maturity, the more convertibles tend to be underpriced. The mispricing

Table 6
Pricing statistics of the binomial-tree model for different moneyness classes

| Moneyness | Mean overpricing (%) | Overpricing std. |
|---|---|---|
| <0.80 | −6.23*** | 0.049 |
| 0.80–0.95 | −2.15*** | 0.034 |
| 0.95–1.05 | −2.21*** | 0.035 |
| 1.05–1.20 | −2.56*** | 0.046 |
| 1.20–2.00 | −1.87*** | 0.039 |
| >2.00 | 0.17 | 0.037 |

*Mean overpricing* states the extent to which market prices are, on average, above model prices for a given moneyness class. ***, ** and * denote significance levels of 1%, 5% and 10%, respectively. *Overpricing std.* is the standard deviation of the observations in the respective class.

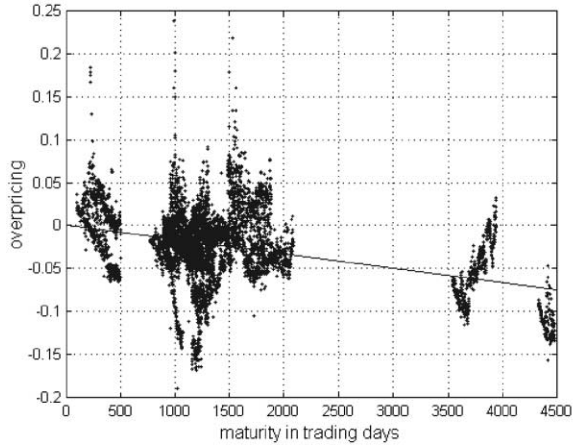


Fig. 3. Maturity/overpricing relationship for the binomial-tree model. This graph shows the percentage overpricing of each daily observed market price measured by the binomial-tree model with respect to the maturity of the bond.

Table 7
Pricing statistics of the binomial-tree model for different maturity classes

| Maturity (trading days) | Mean overpricing (%) | Overpricing std. |
|---|---|---|
| <500 | 0.23 | 0.036 |
| 500–1000 | −1.60*** | 0.030 |
| 1000–1500 | −2.89*** | 0.040 |
| 1500–2500 | −0.71*** | 0.038 |
| >2500 | −6.80*** | 0.039 |

*Mean overpricing* states the extent to which market prices are, on average, above model prices for a given maturity class. ***, ** and * denote significance levels of 1%, 5% and 10%, respectively. *Overpricing std.* is the standard deviation of the observations in the respective class.

is statistically significant with the exception of the class of bonds with the shortest maturity, as can be seen in Table 7. For convertible bonds with a maturity in excess of 2500 days, we detect an underpricing of 6.8% that is statistically significant at the 1% level. This result, however, is caused exclusively by two bonds (Axa 2014 and Axa 2017) as they are the only ones in this class. Interestingly, the underpricing disappears for the class of bonds with the shortest maturity (less than 500 trading days to maturity). The relative ease of implementing arbitrage strategies for short-maturity bonds compared to longer-maturity bonds may be an explanation for this observation. Surprisingly, King (1986) finds a different relationship with increasing mispricing for bonds with a shorter time to maturity.

Overall, our results can be interpreted to support some practitioners' view that convertibles tend to be underpriced by the market. Although we have selected only the most liquid bonds from a complete sample, some of the mispricing may nonetheless be attributable to illiquidity. An alternative explanation for the mispricing may be the rather complex nature of this instrument, making arbitrage strategies costly and sometimes hard to implement. [16] It may therefore take a rather substantial underpricing before an arbitrage strategy can be implemented profitably. This interpretation is supported by our observations that short-maturity bonds, for which arbitrage is easier to implement, and deep in-the-money bonds, which are easier to price, are not underpriced on average.

## 5. Conclusion

We undertake a pricing study for the French convertible bond market. Unlike previous studies in the literature, we do not investigate convertible bond prices on a few specific dates only, but for an entire period of 18 months using daily price data. Moreover, this is the first study modeling the stock price directly instead of using a firm-value model. We propose a binomial-tree model that incorporates embedded options and credit risk, extending existing approaches to be able to account for complex bond characteristics such as embedded call features with various trigger conditions. We find that theoretical values for the analyzed convertible bonds are on average more than 3% higher than the observed market prices. This result applies to both standard convertibles and exchangeable bonds. The majority of the bonds in our sample exhibit a statistically significant mean underpricing. A partition of the sample according to the moneyness indicates that the underpricing is decreasing for bonds that are further in-the-money. This result confirms the findings of other studies. However, unlike previous research, we find a positive relationship between underpricing and maturity. Convertibles with a short maturity are priced more accurately on average, which can plausibly be explained by the difficulty of implementing long-term arbitrage strategies.

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[16] Replication is complicated by the various embedded options. Furthermore, because of the credit risk inherent in convertible bonds, it is not sufficient to replicate a convertible bond with a dynamically adjusted position of the underlying stock. A full replication commonly requires a costly asset swap transaction.

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